AIAA 81-1772R

An Analytical Pilot Rating Method for Highly Elastic Aircraft

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An analytical method was developed to predict pilot ratings for highly elastic aircraft subject to severe mode interactions between rigid body and elastic dynamics. An extension of the standard optimal control model of pilot response was made to include the hypothesis that the pilot controls the system with an internal model consisting of the slowly varying part of the aircraft dynamics. This modified optimal control model was analytically evaluated for a longitudinal pitch tracking task on a large, flexible aircraft. Parametric variations in the undamped natural frequencies of two symmetric elastic modes were made to induce varying amounts of mode interaction. The model proved successful in discriminating when the pilot can or cannot visually separate rigid from elastic pitch response in the turbulence-excited tracking task. This method shows considerable promise in making it possible to investigate such mode interaction effects on handling qualities in the preliminary design stage of new aircraft.

Introduction

THE handling or flying qualities of a piloted aircraft are the static and dynamic characteristics that influence the ease and precision with which a pilot is able to perform the control task required in support of the aircraft mission flight phase. Thus the handling qualities depend not only on aircraft characteristics and a mission flight phase, but also on the pilot's subjective opinion of the ease with which he can perform the control task.

To accurately assess the pilot's opinion of the handling qualities of an aircraft prior to first flight of a prototype, a ground-based simulation is usually required. In the early stages of the design, it is more economical to use a mathematical pilot-modeling simulation because the design parameters can be easily adjusted. The pilot's assessment is then related to some scale, such as the widely accepted Cooper-Harper pilot rating scale.[1]

Much research has been done to determine the relations between the parameters of the rigid body, small perturbation equations of motion and the pilot rating. The handling qualities requirements for a rigid airplane in Ref. 2 are typical results of such research. Most airplanes of the past have been relatively rigid, such that the elasticity of the airplanes do not contribute significantly to the pilot-perceived handling qualities.

Recent advances in control-configured vehicles' design and active control technology make it possible to increase aircraft size and the utilization of lighter structures in future designs. The elastic behavior of these vehicles will therefore become an appreciable influence on their handling qualities. Because of the potential adverse effects of elastic mode interaction with the rigid body dynamics, there is a need for handling qualities assessment of this effect in the preliminary design phase of new airplanes.

It is known that static aeroelastic deflections of an aircraft structure modify the aerodynamic pressure distributions, which results in stability derivative changes associated with the rigid body, small perturbation equations of motion. Early attempts to account for aeroelastic effects on aircraft stability and control took the approach of making static aeroelastic corrections to the aerodynamic stability derivatives.[3-5]

For flying in high dynamic pressure environments, such as terrain following in turbulent air, the dynamic effects of flexibility are important enough that they must be included as additional degrees of freedom. A common approach has been to approximate the dynamics by a truncated set of superimposed orthogonal vibration modes. In this case, the phenomena of most interest are the effects of aerodynamic coupling between the various elastic modes and between elastic and rigid body modes, as well as elastic mode interaction with the feedback control system. Reference 6 was one of the earliest comprehensive studies of this problem; more recent comprehensive work is documented in Ref. 7.

The handling qualities parameters, such as phugoid, short-period, and dutch-roll frequencies and damping ratios, which have been determined pertinent to rigid airplanes, are less useful for a flexible airplane with elastic mode frequencies close to the rigid body frequencies. When multiple frequencies are in proximity to one another, the pilot cannot easily discern individual modes of motion; rather his opinion of the transient dynamics will likely be based on the time history of the total motion. No performance criteria suitable for handling qualities specification are presently available for such higher-order responses.

It seems quite possible that the desirable ranges of parameter values could be significantly affected by elastic mode degrees of freedom, particularly when some of the modes have natural frequencies of the same order of magnitude as the frequencies of the rigid body alone. It is not at all clear that the handling qualities should be specified by rigid body dynamic parameters when such mode interaction is present. In fact, the pilot could not tell, for example, how much of a given pitch angle response to command input is due to rigid body and how much to low-frequency elastic modes.

The key in developing handling qualities criteria and eventually specifications for severe mode interaction situations is to establish when and under what conditions the pilot can visually separate the rigid body response from the total response. In conditions when he cannot, a structural mode suppression control system probably will be required.

The primary objective of our work was to develop an analytical method to determine when the pilot can visually separate the rigid body motion from the total motion and when he cannot, in terms of the small perturbation equations of motion parameters. This study is an extension of the experimental work done in Ref. 8, where a ground-based, pilot-flown simulation was used. The mathematical pilot modeling

Presented as Paper 81-1772 at the AIAA Guidance and Control Conference, Albuquerque, N. Mex., Aug. 19-21, 1981; submitted Sept. 30, 1981; revision received April 5, 1982.

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simulation approach was used to assess the effects of mode interaction on the pilot opinion rating. An extension of the optimal control model for the human pilot[9] was made to assess the mode interaction effects.

Complete details of the work described herein are contained in Ref. 10. Schmidt[11] has formulated an alternative approach which appears easier to use than, the one presented here. Still other reduced-order internal model applications of the optimal control model are discussed in Ref. 12.

Aircraft Dynamics

The equations of motion of an elastic airplane consist of the six rigid body degrees of freedom plus vertical and lateral elastic local structure deformations due to its inherent flexibility. The elastic motion is expressed in terms of symmetric and antisymmetric, orthogonal, lumped mass mode shapes and generalized coordinates, where the airplane is idealized to horizontal (xy plane) and vertical (xz plane) platelike structures in the mode shape calculations.

In this work, only a longitudinal pitch tracking task is used with two low-frequency symmetric elastic modes plus the three longitudinal rigid body degrees of freedom. The aircraft is excited by an atmospheric turbulence shaping filter model derived from the Dryden gust power spectra which have $u_g(t)$, $\alpha_g(t)$, and $q_g(t)$ components. The resulting state vector dynamics includes eight aircraft and four gust state variables.

$$\dot{x}_a(t) = A_a x_a(t) + B_a u_a(t) + E_a w_a(t) \tag{1}$$

where

$$x_a(t) = [u_g, \alpha_{g_1}, \alpha_g, u, \alpha, \theta, \dot{\theta}, \xi_1, \xi_2, \dot{\xi}_1, \dot{\xi}_2]'$$

$$u_a(t) = \delta_e(t) \equiv \text{elevator deflection}$$

and $w_a(t)$ is a (4×1) vector with independent, zero mean, Gaussian white noise state variables.

The total pitch angle that the pilot feels and sees, either on the outside horizon or the attitude indicator display, is given by Eq. (2), where $\theta_e(t)$ is the elastic contribution to the total pitch angle at the cockpit location.

$$\begin{aligned} y_a(x_p,t) &= \theta(t) - \sum_{i=1}^{n} \phi_j'(x_p)\xi_j(t) \\ &= \theta(t) - \theta_e(x_p,t) = C_a x_a(t) \end{aligned} \tag{2}$$

Pilot Model Assumptions

The fundamental assumption in the model is that a well-motivated, well-trained pilot acts in a near-optimal manner subject to his internal limitations and understanding of the task. The model adapts to task specifications and requirements automatically. Thus for a new situation, the optimal control model can be modified by just determining the operative limitations and the new control task. A review of past model applications is given in Ref. 13.

The information processor in the pilot model operates on a noisy, delayed version of the displayed variables to obtain a best estimate of the aircraft state vector. This is accomplished by a Kalman filter and a least mean square predictor and makes use of an internal model.[14] The internal model of the pilot may be considered to consist of 1) knowledge of the aircraft dynamics and the control inputs, 2) a statistical knowledge of the disturbances acting on the aircraft and how they effect it, and 3) knowledge of the task to be performed.

In many instances the assumption that the internal model is an exact replica of the system dynamics model, that is, a perfect internal model, appears to be satisfactory. However, in a highly complex system with a large number of state variables, with a single display it is unlikely to be modeled perfectly by the pilot.

The experimental results of Ref. 8 showed that the pilot action in a pitch tracking task closely resembled that of controlling rigid body error rather than the total pitch error displayed to him, which included higher-frequency elastic contributions to the response. This indicated the pilot's ability to filter out a sufficiently high-frequency oscillation and leads to our hypothesis that the pilot uses the slowly varying part of the total system dynamics as his internal model when tracking rigid body pitch commands. When the frequencies were close, his tracking was much worse.

The separation of the slowly varying part from the total system dynamics is accomplished by use of a singular perturbation method.[15]

$$\begin{aligned} \dot{x}_1(t) &= A_{11}x_1(t) + A_{12}x_2(t) + B_1 u_a(t) + E_1 w_a(t) \\ \mu\dot{x}_2(t) &= A_{21}x_1(t) + A_{22}x_2(t) + B_2 u_a(t) + E_2 w_a(t) \\ y_a(t) &= C_1 x_1(t) + C_2 x_2(t) \end{aligned} \tag{3}$$

where $x_1(t)$ is the rigid body and gust shaping filters state vector; $x_2(t)$ the elastic modes state vector; and μ a small positive parameter inversely proportional to elastic mode frequencies that can be left an unknown in this analysis.

Then, by letting $\mu \to 0^+$,

$$\begin{aligned} \dot{x}_d(t) &= A_d x_d(t) + B_d u_a(t) + E_d w_a(t) \\ y_d(t) &= C_d x_d(t) + D_d u_a(t) + F_d w_a(t) \end{aligned} \tag{4}$$

where

$$A_d = A_{11} - A_{12}A_{22}^{-1}A_{21}$$

$$B_d = B_1 - A_{12}A_{22}^{-1}B_2$$

$$C_d = C_1 - C_2A_{22}^{-1}A_{21}$$

$$D_d = -C_2A_{22}^{-1}B_2$$

$$E_d = E_1 - A_{12}A_{22}^{-1}E_2$$

$$F_d = -C_2A_{22}^{-1}E_2$$

The pilot perceives $y_p(t)$, which is a noisy version of $y_a(t)$ as shown in Fig. 1. His internal model of the slowly varying dynamics of Eq. (4) is mathematically accomplished in the Kalman estimator.

In many instances, the assumption that the internal model is an exact replica of the system model, i.e., perfect internal model, appears to be a satisfactory one. There are situations in which the assumption of a perfect internal model does not appear tenable. In a highly complex system, i.e., one with a large number of state variables, with a single display it is unlikely to be modeled perfectly by the pilot.

By using an imperfect internal model, the computational task is increased, since it involves the solution of a matrix delay differential equation. Other than the pure time delay τ, which we approximate by a first-order Padé polynomial, the pilot has inherent limitations of perceptual noise and observation thresholds on displayed information.

An important assumption about the optimal control pilot model is that the pilot's control task is adequately reflected in the choice of a control $r(\cdot)$ that minimizes the cost functional of the form

$$J(r) = \lim_{T\to\infty} E\left\{\frac{1}{T}\int_0^T [y_a'(t)Q_y y_a(t) + \dot{r}'(t)Q_R\dot{r}(t)]\mathrm{d}t\right\} \tag{5}$$

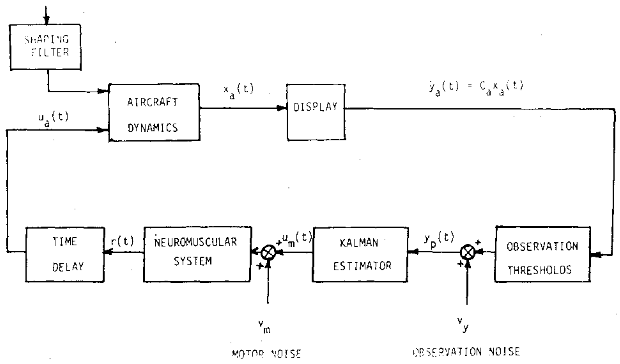

Fig. 1 Modified optimal control model of human pilot response.

conditioned on the perceived information $y_p(\cdot)$. Q_y is a specified constant, symmetric, nonnegative definite matrix which depends on the task specification. The selection of the weighting $Q_y = \text{diag}\,[q_{y_i}]$ is such that

$$q_{y_i} = \left| \frac{1}{y_{p_{i,\max}}} \right|^2$$

where $y_{p_{i,\max}}$ is the maximum desired or allowable value of y_{p_i} (5 deg of pitch in this work). The weighting $Q_R = \text{diag}[q_{r_i}]$ is a positive definite matrix and is not specified before the pilot model equations are solved. The control rate term is used to account for the pilot's limitation on rate of control motion and introduces first-order neuromuscular lag dynamics in the pilot model. A matrix T_n is assumed to be in the form $T_n = \text{diag}[t_{n_i}]$, $i = 1,2,\ldots$. The scalars t_{n_i} are neuromuscular time constants of human limbs which have typical values of 0.1 s for single-axis tasks, independent of the system to be controlled. Thus the weighting q_{r_i} are adjusted iteratively until each $t_{n_i} \simeq 0.1$ s. If the resulting q_{r_i} weighting is such that $1/\sqrt{q_{r_i}}$ is much greater than the physical rate at which one can move control r_i, then $q_{r_i} = |1/\dot{r}_{i,\max}|^2$ must be used.

Pilot Model Solution

Augmenting the aircraft dynamics Eq. (1) with the additional states and dynamics due to the pilot's neuromuscular lag and reaction time delay gives

$$\dot{x}_c(t) = A_c x_c(t) + B_c u_m(t) + E_c w_c(t), \quad y_a(t) = C_c x_c(t) \tag{6}$$

where

$$x_c = [x_a, z, r]', \qquad B_c = [0, 0, T_n^{-1}]'$$

$$w_c = [w_a, v_m]', \qquad C_c = [C_a, 0, 0]$$

$$A_c = \begin{bmatrix} A_a & B_a & -B_a \\ 0 & -2/\tau & 4/\tau \\ 0 & 0 & -T_n^{-1} \end{bmatrix}, \quad E_c = \begin{bmatrix} E_a & 0 \\ 0 & 0 \\ 0 & T_n^{-1} \end{bmatrix}$$

z is the first-order Padé approximation state variable. Equation (6) is the "actual" dynamics to be controlled by the pilot.

The pilot's control input $r(t)$ that minimizes $J(r)$ is generated based on the internal model Eq. (4) and the pilot's delay states

$$\begin{aligned} \dot{x}_s(t) &= A_s x_s(t) + B_s r(t) + E_s w_a(t) \\ y_s(t) &= C_s x_s(t) + D_s r(t) + F_s w_a(t) \end{aligned} \tag{7}$$

where

$$x_s = \begin{bmatrix} x_d \\ z \end{bmatrix}, \quad A_s = \begin{bmatrix} A_d & B_d \\ 0 & -2/\tau \end{bmatrix}, \quad B_s = \begin{bmatrix} -B_d \\ 4/\tau \end{bmatrix}$$

$$C_s = [C_d, D_d], \quad D_s = [-D_d], \quad F_s = [F_d], \quad E_s = \begin{bmatrix} E_d \\ 0 \end{bmatrix}$$

The command control of the pilot is given by

$$\begin{aligned} u_m(t) &= -L_{\text{opt}} x_s(t) \\ &= -L^* x_t(t) \end{aligned} \tag{8}$$

where

$$L^* = [L_{\text{opt}} 0], \qquad x_t = \begin{bmatrix} x_s \\ r \end{bmatrix}, \quad T_n = P_{22}^{-1} Q_R$$

$$L_{\text{opt}} = P_{22}^{-1} P_{12}', \qquad P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}' & P_{22} \end{bmatrix}$$

satisfies the equation

$$A_0' P + P A_0 + C_0' Q_y C_0 - P B_0 Q_R^{-1} B_0' P = 0 \tag{9}$$

where

$$A_0 = \begin{bmatrix} A_s & B_s \\ 0 & 0 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad C_0 = [C_s, D_s]$$

The state $\hat{x}_t(t)$ is the best estimate of $x_t(t)$ generated by a Kalman filter

$$\dot{\hat{x}}_t(t) = A_t \hat{x}_t(t) + B_t u_m(t) + K[y_p(t) - C_0 \hat{x}_t(t)] \tag{10}$$

where

$$K = \Sigma C_0' V_y^{-1} \tag{11}$$

and Σ satisfies the equation

$$A_t \Sigma + \Sigma A_t' + E_0 W_t E_0' - \Sigma C_0' V_y^{-1} C_0 \Sigma = 0 \tag{12}$$

where

$$A_t = \begin{bmatrix} A_s & B_s \\ 0 & -T_n^{-1} \end{bmatrix}, \quad B_t = \begin{bmatrix} 0 \\ T_n^{-1} \end{bmatrix}$$

$$E_0 = \begin{bmatrix} E_s & 0 \\ 0 & T_n^{-1} \end{bmatrix}, \quad W_t = \begin{bmatrix} W & 0 \\ 0 & V_m \end{bmatrix}$$

Combining Eqs. (6), (8), and (10) yields the closed-loop system

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) - B_c L^* \hat{x}_t(t) + E_c w_c(t) \\ \dot{\hat{x}}_t(t) &= (A_t - B_t L^*) \hat{x}_t(t) + K[C_c x_c(t) - C_0 \hat{x}_t(t) + v_y(t)] \end{aligned} \tag{13}$$

or

$$\dot{\psi} = F\psi + Gw \tag{14}$$

where $y_p(t) = y_a(t) + v_y(t)$,

$$\psi = \begin{bmatrix} x_c \\ \hat{x}_t \end{bmatrix}, \quad F = \begin{bmatrix} A_c & -B_c L^* \\ KC_c & (A_t - B_t L^* - KC_0) \end{bmatrix}$$

$$G = \begin{bmatrix} E_c & 0 \\ 0 & K \end{bmatrix}, \quad w = \begin{bmatrix} w_c \\ v_y \end{bmatrix}$$

Thus,

$$\text{cov}\psi = \begin{bmatrix} \text{cov}x_c x_c' & \text{cov}x_c \hat{x}_t' \\ \text{cov}\hat{x}_t x_c' & \text{cov}\hat{x}_t \hat{x}_t' \end{bmatrix} \equiv \Psi \tag{15}$$

is the solution of

$$\dot{\Psi} = F\Psi + \Psi F' + G\Omega G' \tag{16}$$

where

$$\Omega = \begin{bmatrix} W_c & 0 \\ 0 & V_y \end{bmatrix}$$

with W_c and V_y the autocovariance matrices of $w_c(t)$ and $v_y(t)$.

Pilot Opinion Rating Technique

Hess[16] has formulated a pilot rating technique for the optimal control pilot modeling procedure. The technique has been successfully validated in a variety of tasks.[16,17] The rating technique can be stated as follows: If

1) the index of performance and model parameters in the optimal control pilot modeling procedure yield a dynamically representative model of the human pilot,

2) the variables selected for inclusion in the index of performance are directly observable by the pilot,

3) the weighting coefficients in the index of performance are chosen as the squares of the reciprocals of maximum "allowable" deviations of the respective variables, and these deviations are consonant with the task as perceived by the pilot.

Then, the numerical value of the index of performance resulting from the modeling procedure can be related to the numerical pilot rating which the pilot assigns to the vehicle and task.

Schmidt[17] developed Eq. (17) as an empirical fit to Hess's data. Results by others since our work was completed indicate the constant 0.3 in Eq. (17) should be dropped, or left undetermined, and the equation used to predict relative ratings, which is how it was used in our work. Thus, our results would be unaffected.

$$\text{POR} \simeq 2.5\, \ell n(10\, J) + 0.3 \tag{17}$$

where

POR = pilot opinion rating on Cooper-Harper scale

J = value of the performance index

Model Evaluation

Evaluation of the modified optimal pilot model was conducted for an early version of the B-1 aircraft dynamics at a Mach 0.85 sea level flight condition. Ten open-loop cases listed in Table 1 were used where parametric reductions were made in the undamped natural frequencies of the two elastic modes ω_1 and ω_2. Case 1 is the original bare airframe dynamics from which the changes were made. Structural damping ratios for ζ_1 and ζ_2 were assumed to have values of 0.02 each.

The lowering of the elastic mode natural frequencies resulted in mode interaction, which lowered the coupled short-period and phugoid frequencies or made one of them split into positive and negative real roots. A full state feedback control law was used to place the roots of the closed-loop characteristic equation at precise values for each case. The rigid body dynamics were always maintained the same as case 1 and the elastic mode coupled frequencies, ω_{1e} and ω_{2e}, were placed at the values of Table 1 for each case. This ensured that the pilot ratings were based on the relative amplitudes of rigid and elastic pitch angle responses and not on poor rigid body dynamics when the pilot model was included.

The results of using the standard optimal control model for the human pilot and the modified model are shown in Tables 2 and 3, respectively. They are normalized to a POR of 1 for case 1. These results clearly indicate that when there is severe mode interaction, such as cases 3, 6, and 9, the standard OCM gives very low pilot ratings. In contrast, the modified OCM gives higher ratings since it includes the visual separation process the pilot tries to use to discern rigid body response from the elastic response. When the interaction is small, both models give nearly the same pilot ratings. In all cases of relatively poor ratings (2, 3, 5, 6, 8, and 9), the coupled first elastic mode damping ratio ζ_{1e} was 0.1 or greater. This ap-

Table 1 Natural frequencies and damping ratios of open-loop cases

Case	ω_1, rad/s	ω_2, rad/s	ω_{ph}, rad/s	ζ_{ph}	ω_{sp}, rad/s	ζ_{sp}	ω_{1e}, rad/s	ζ_{1e}	ω_{2e}, rad/s	ζ_{2e}
1	13.59	21.18	0.0665	0.0312	2.9334	0.5209	13.236	0.0497	21.395	0.02112
2	8	21.18	0.04614	0.001376	2.581	0.4992	7.508	0.1127	21.390	0.02104
3	4	21.18	0.1412	0.1336	Real roots +1.652 −2.266		4.544	0.3892	21.380	0.02102
4	13.49	15.00	0.06345	0.02899	2.889	0.5247	13.04	0.04508	15.480	0.03052
5	8	15.00	0.04489	0.001946	2.586	0.5031	7.400	0.1073	15.340	0.02787
6	4	15.00	0.1411	0.134	Real roots +1.568 −2.172		4.356	0.3990	15.320	0.02761
7	13.59	13.59	0.06203	0.02797	2.87	0.5262	12.76	0.03821	14.380	0.02923
8	8	13.59	0.04433	0.002208	2.588	0.5048	7.345	0.1042	13.990	0.03117
9	4	13.59	0.1411	0.1350	Real roots +1.527 −2.126		4.269	0.4035	13.970	0.03062
10	8	8	0.03801	0.005574	2.608	0.5245	6.403	0.04999	9.390	0.08138

Table 2 Performance prediction by the standard OCM

Case	θ_{rms}, deg	$\dot{\theta}_{rms}$, deg/s	δ_{rms}, rad × 10^3	POR
1	0.3895	1.650	1.143	1.0
2	0.5422	2.343	2.475	2.6
3	0.5042	1.573	2.234	1.0
4	0.3612	0.917	0.910	1.0
5	0.5096	2.335	2.266	2.5
6	0.4604	1.721	2.061	1.2
7	0.3437	1.127	1.392	1.0
8	0.4955	2.288	2.142	2.4
9	0.4445	1.750	2.000	1.2
10	0.3833	1.591	2.810	1.0

Table 3 Performance prediction by the modified OCM

Case	θ_{rms}, deg	$\dot{\theta}_{rms}$, deg/s	δ_{rms}, rad × 10^3	POR
1	0.3663	1.848	0.829	1.2
2	0.5724	2.832	1.199	3.3
3	0.8123	3.721	1.014	4.7
4	0.3157	0.999	1.024	1.0
5	0.5345	2.746	1.195	3.2
6	0.7374	3.470	0.872	4.3
7	0.3228	1.428	0.869	1.0
8	0.5172	2.657	1.181	3.0
9	0.7043	3.360	0.833	4.1
10	0.4187	2.091	1.167	1.8

parently makes it more difficult for the pilot to separate short-period from elastic mode 1. Case 10 was better because ζ_{1e} was low.

Concluding Remarks

The modified optimal control pilot model will allow the investigation of mode interaction effects on aircraft handling qualities at an early stage in preliminary design before ground-based simulator data are available. If such analyses indicated the handling qualities are severely affected by elastic mode interaction with rigid body dynamics, an elastic mode suppression control system would likely need to be designed.

Only one longitudinal flight condition was used in the evaluation of the model. The model validity for different flight conditions and for other classes of aircraft needs to be confirmed. Also, the method does show encouraging qualitative trends, but experimental validation should be done in future research.

Acknowledgment

This research was partially supported by the Vehicle Dynamics and Control Division, NASA Dryden Flight Research Center under Grant NSG 4018.

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